

(i) Find out the unique solution satisfying the initial condition $y(0) = 2$, and sketch the graph of this function.

(ii) Verify that $y = 1$ is a constant solution and determine whether it is stable or unstable.

5. A hot cup of coffee is served outside where the temperature is 10°C , and will be cooling according to the Newton's law

$$T' = -k(T - T_e)$$

where $T(t)$ is the temperature of the coffee and T_e the external temperature.

Assume that initially the coffee is at 80°C and that after 1 minute its temperature decreased up to 60°C . How much time should you wait before you can drink the coffee at the perfect temperature of 40°C ?

6. The height of some plant is growing at the rate of

$$H'(t) = \frac{t^2+1}{3-t} \quad (\text{meters per year})$$

Find out how much the height of the plant increased during the first year.

7. Consider the function

$$f(x, y) = x \ln(xy^2) - 2x + \frac{x}{y^2}$$

Determine the domain of the function, find all critical points and determine whether they are local (or global) maxima or minima, or whether they are saddle points.

8. A firm produces according to the Cobb-Douglas function

$$P(x, y) = 160 x^{3/4} y^{1/4}$$

whenever x units of capital and y units of labor are allocated to production.

(i) Draw the curves of constant production $P(x, y) = c$ (otherwise said, the level curves of the function) for different values of c (e.g. $c = 1, c = 2, c = 3, \text{etc} \dots$)

(ii) Suppose each unit of capital costs 5.000 euro and each unit of labor costs 3.500 euro. If the manufacturer has 800.000 euro to spend, how many units of capital and labor should be allocated to maximize the production?