

Omigosh . . .

Exercise 1

First-time investor Billy Joe Dupree pays \$15,000 into a savings plan that runs over 12 years at an annual interest rate of 5% (simple interest). Five years into the savings plan (= five years later), Billy Joe learns from his brother Cooper that compound interest usually yields higher returns than simple interest. He therefore thinks about withdrawing his money (= \$15,000 plus interest accrued so far) from the savings plan and re-investing everything at a compound annual interest rate of 3.5% for the remaining seven years. Will he thus actually have a higher future value after the full twelve years? In other words: Should he really switch after five years or rather stay with his original plan?

Exercise 2

In the meantime, Billy Joe's brother Cooper is looking forward to his retirement. He intends to invest his lifetime savings of \$180,000 into an annuity due at 5% compound annual interest rate that runs over 25 years.

- a) What annual payments can he expect from this annuity?
- b) Would an ordinary annuity (all other parameters identical) generally yield higher annual payments? Explain!
- c) Cooper is very disappointed about the result in a) as he had expected much higher annual payments. He therefore decides to keep on working for five more years and invest those \$180,000 into a deferred annuity due at a compound annual interest rate of 5% that provides 20 annual payments after five years have passed. What annual payments can Cooper now expect for his retirement?

Exercise 3

Millionaire Richard W. Bullford III wants to further the fine arts and therefore endows the Richard W. Bullford III Fine Arts Foundation with a total of \$500,000. The money will be invested into a perpetuity due at a compound annual interest rate of 4%.

- a) How much money can the foundation use annually to promote the fine arts?
- b) The result of a) obviously depends on two input factors: the amount of money donated by Richard W. Bullford III and the interest rate. Which of these input factors would have a stronger impact on the annual amount of money available for the fine arts if it increased slightly? Use the concept of elasticity to find an answer.

Exercise 4

Potato chips producer Crisp&Crunchy is desperate to cut costs to survive in an increasingly competitive market. Controller Randy suggests an increase in daily production from its current 8 tons to 9 tons as this would lower average costs. The company's current cost situation is modelled by

$$TC(Q) = Q^3 - 20Q^2 + 500Q + 2,000 .$$

- Do you agree with Randy? Find some convincing mathematical argument.
- For what daily output in tons would the company minimize its average variable costs? What total costs would the company report for this very output?

Exercise 5

According to product manager Leroy Smith's latest finding, demand and supply for his company's premier kitchen tissue (Soft&Swift) are governed by:

$$P_D(Q_D) = 100/(Q_D + 2) \text{ (demand)} \quad \text{and} \quad Q_S(P_S) = 0.25 \cdot P_S + 5.5 \text{ (supply)}.$$

- What revenue would Leroy find in the market equilibrium?
- Calculate the elasticity of demand with respect to price for a price of $P_D = 12$. What happens with this very elasticity for large values of P_D ?

Exercise 6

HR manager Bill Cruel has observed that employee satisfaction essentially depends on two factors: salary (measured in \$ and indicated by S) and the number of days off granted (indicated by D). Employee motivation M is best modelled by:

$$M(S,D) = 0.001 \cdot S^3 \cdot D^{1.5} .$$

- Find some sound mathematical argument to demonstrate that any increase in either S or D would always have a positive impact on motivation.
- Bill is willing to either generally raise salaries by 2.5% or increase the number of days off from 20 to 21. Which measure would have the strongest impact on employee satisfaction? Use the concept of differentials to find an answer.
- Find the level curve that expresses S as a function of D for a motivation of 1,000,000. Explain the meaning of this level curve.