Getting ready for the real thing . . .

Exercise 1

Farmer Joe has found out that his corn yield is directly linked to the amount of fertilizer he pours on his vast fields, located just off the famous Route 66 in rural Illinois. His in-depth mathematical analysis reveals that

$$Y(F) = (F + 1)^2 \cdot e^{1-F}, \quad F \geq 0,$$

by and large describes his corn yield as a function of the amount of fertilizer used (both F and Y given in appropriate units).

a) What corn yield would Joe find if he used no fertilizer at all?

b) What maximum corn yield can Joe hope to find if he were allowed to use any (positive) amount of fertilizer?

In addition, Joe has discovered that his cost situation (at least the part attributable to fertilizer) is best described by

$$TC(F) = 12.5 + 2.5F^2,$$

again assuming only positive values for F.

c) What value would you find for the elasticity of TC with respect to F if Joe poured more and more fertilizer on his fields? Make sure you interpret your findings.

Exercise 2

Manager Bill is desperate. No matter how much capital and labor he invests into his $500 million business, his total output seems to shrink more and more. Management accountant Steve analyzes the situation and comes up with the following production function:

$$Q(L,K) = (1 + LK)^{-1}, \quad \text{where both } L \geq 0 \text{ and } K \geq 0.$$

a) Verify mathematically that Bill's observation is true.

b) What output would Bill's company report if he invested no capital at all, yet used 1,000 units of labor? Fairly easy . . .

c) Find the isoquant L(K) for an output of 50.

Exercise 3

Many years of experience have shown marketing manager Lori that consumer demand Q for her company's no.1 product is best described by (yup, there's a logarithm involved)

$Q(P_A, P_B, Y) = 10 - 2.5P_A + 5\ln(2 + P_B) + 0.003Y$.

The company currently charges a standard price of $P_A = \$50$ per unit, while its major competitor charges $P_B = \$55$ per unit. Current consumer income is reported to be $Y = \$50,000$. Which of the three input variables would have the greatest impact (whether positive or negative) on consumer demand Q? Use the elasticity concept to find an answer.

Exercise 4

A bit of financial math that may sound familiar: How long does it take to double a given principal $P_0 > 0$ at a compound annual interest rate of 10%?

Exercise 5

And some more financial math: Given the present value of an ordinary perpetuity (a_∞), find the elasticity of this very present value with respect to the interest rate i. What does your result imply?

Exercise 6

And another one: Lucky Bill has won a lifelong annuity of $48,000 annually in the lottery. He is currently 47 years old and has a life expectancy of 31 years. The first payment is, of course, due right away. His friend Cory has won the grand prize of $1,000,000 in the same lottery.

Who would you say is more fortunate assuming that both persons can invest money at a fixed annual interest rate of 4%?